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TECHNICAL NOTE 3160

A CALCULATION STUDY OF WING-AILERON FLUTTER IN TWO DEGREES OF FREEDOM FOR TWO-DIMENSIONAL SUPERSONIC FLOW

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A CALCULATION STUDY OF WING-AILERON FLUTTER IN TWO DEGREES OF FREEDOM FOR TWO-DIMENSIONAL SUPERSONIC FLOW

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SUMMARY

Results are presented of some sample calculations made for the bending-aileron and torsion-aileron flutter of an idealized wing-aileron system (infinite aspect ratio, full-span aileron hinged at its leading edge) for several values of Mach number in supersonic flow and also for some subsonic Mach numbers. It is emphasized that the results are subject to various limitations and are not intended to be applicable to particular configurations. They may serve, however, to provide preliminary knowledge of the influence of variations in certain parameters and to indicate some of the differences in the trends exhibited by the calculations for high speeds and those trends known to exist for low speeds.

The more influential parameters appear to be: mass balance of the control surface, control-surface frequency relative to wing frequency, and structural damping. An interesting result is the indication of an apparent reverse effect of mass balance at the transonic speeds for some wing-aileron configurations. A high value of control-surface frequency appears beneficial as does structural damping of the system. In several of the calculations, the possibility of flutter in a single degree of freedom is indicated.

INTRODUCTION

Numerous investigations, both analytical and experimental, of the problem of flutter involving control surfaces have been made in order to determine what parameters influence this type of flutter and to find means of preventing it. These investigations have been concerned chiefly with flutter at low speeds. The present paper aims at contributing to the subject through calculations made by analytical consideration of the supersonic speed range. Since the analysis has been idealized and has many limitations, some remarks emphasizing the nature of the limitations follow this introduction and the list of symbols.

Some of the factors affecting control-surface flutter at low speeds have long been known. Even before the development of the field of unsteady aerodynamics, some conclusions had been reached by Von Baumhauer and Koning (ref. 1) on the influence of control-surface mass balance. Similar conclusions were reached by Frazer and Duncan in reference 2. Later investigations of control-surface flutter include the treatment of Theodorsen and Garrick (ref. 3), where the results are presented of a systematic analytical survey of the effects of various parameters on the flutter characteristics of a wing-aileron system in incompressible flow. Other investigations have been made by Van de Vooren (ref. 4), numerous British investigators (see, for example, ref. 5), Dorr (ref. 6), and others. This work has indicated the critical dependence of control-surface flutter on such parameters as control-surface unbalance, structural damping, and the relative stiffnesses in wing bending, wing torsion, and aileron motion.

The criteria determined analytically for the case of incompressible flow have generally been carried over, together with experience, to form the basis for criteria for the prevention of control-surface flutter in the subsonic case. Although subsonic aerodynamic coefficients relating to the control surface have been available in, for example, reference 7, in certain British sources, and more recently in more extensive form in reference 8, no systematic calculations using these coefficients have been published.

With the attainment of supersonic speeds in flight the question of the applicability of standards set for the subsonic case to the supersonic case has become increasingly important. Little practical experimental knowledge exists and until recently no theoretical aerodynamic coefficients relating to control-surface flutter at supersonic speeds were available. These coefficients have now been tabulated for a wing in two-dimensional supersonic flow in reference 9 as an extension of reference 10. Coefficients for the sonic case have been developed and tabulated in reference 11.

In the present paper the aerodynamic coefficients of references 9, 10, and 11 have been employed with the analytical approach of reference 10 in the performance of some sample calculations for wing-aileron flutter in two degrees of freedom at supersonic speeds. For comparison, a few examples for subsonic flow are included. The investigation applies to an infinite-aspect-ratio wing-aileron system with an aileron hinged at its leading edge and is not concerned with any existing configuration. Control-surface configurations for supersonic flight are not yet standardized and hence the investigation has been limited to an idealized system with the aim of gaining some indications of what to look out for and what to expect in control-surface flutter at supersonic speeds as compared with what is known for the subsonic case. It is realized that similar calculations may exist elsewhere in unpublished form and, further, that a solution to the complete problem will require the consideration of many factors not included in the present work.

SYMBOLS

b	one-half chord of wing-aileron system
c	velocity of sound
g_h, g_α, g_β	structural damping coefficients
h	vertical displacement of axis of rotation
I_α	moment of inertia of wing-aileron combination about elastic axis per unit span length
I_β	moment of inertia of aileron about x_1 (hinge axis) per unit span length
m	mass of wing-aileron combination per unit span length
M	Mach number
L_n, M_n, N_n $(n = 1,2,\ldots 6)$	aerodynamic coefficients of lift and moment (see refs. 9 and 10)
r_α	radius of gyration of wing-aileron combination referred to x_o, $\sqrt{I_\alpha/mb^2}$
r_β	reduced radius of gyration of aileron referred to x_1, $\sqrt{I_\beta/mb^2}$
S_α	static moment of wing-aileron combination per unit span length referred to x_o
S_β	static monent of aileron per unit span length referred to x_1
v	speed of forward motion (or velocity of main stream)
x_o	coordinate of elastic axis measured from leading edge of wing, referred to chord 2b as reference length
x_1	coordinate of aileron hinge measured from leading edge of wing, referred to chord 2b as reference length
x_α	location of center of gravity of wing-aileron combination referred to x_o

x_β reduced location of center of gravity of aileron referred to x_1, S_β/mb

α angular displacement about rotation point

β angular displacement of aileron about aileron hinge

μ mass ratio parameter, $m/4\rho b^2$

ω circular frequency at flutter

ω_h natural circular frequency of wing in vertical translation

ω_α natural circular frequency of torsional vibrations about elastic axis

ω_β natural circular frequency of torsional vibrations of aileron about x_1

ρ mass density of air

$$\Omega_h X = \mu(\omega_h/\omega)^2(1 + ig_h)$$

$$\Omega_\alpha X = \mu r_\alpha{}^2(\omega_\alpha/\omega)^2(1 + ig_\alpha)$$

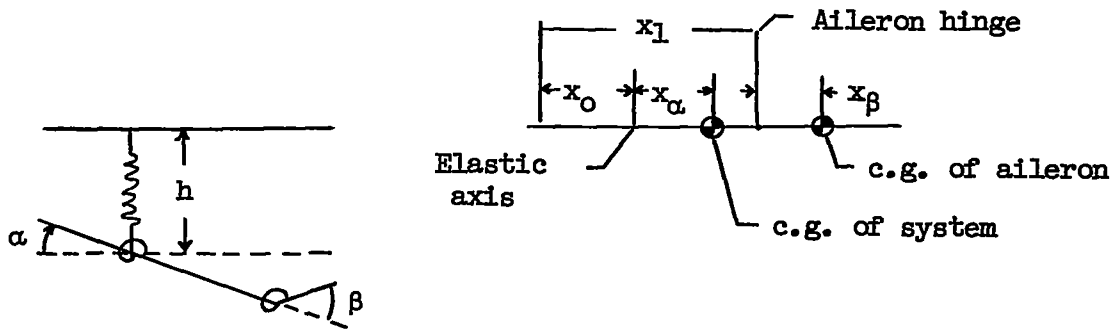
$$\Omega_\beta X = \mu r_\beta{}^2(\omega_\beta/\omega)^2(1 + ig_\beta)$$

REMARKS EMPHASIZING LIMITATIONS

The calculations discussed in this report were undertaken in order to gain some insight into the problem of flutter involving control surfaces at supersonic speeds, a field in which very few published results exist, and were intended in part to serve as a possible guide to experimental investigations of the problem. It is felt that the results of the numerical calculations might be of interest to other investigators. It should be clearly kept in mind that the results presented are subject to various limitations and are not intended for application to an actual configuration. As indicated in the introduction, the calculations were performed for the case of two-dimensional flow because of the availability of the coefficients and were applied to a highly idealized wing-aileron system (infinite aspect ratio, full-span aileron hinged at its leading edge). Parameter variations treated are rather limited. The results may be considerably altered by such factors as finite span, thickness, additional degrees of freedom, and other parameters which were not considered. The results should serve, however, to provide a preliminary knowledge of the influence of variations in certain parameters and to indicate some of the differences in trends exhibited by the calculations for high speeds and the trends known to exist for low speeds.

SYSTEM CONSIDERED AND METHOD OF ANALYSIS

Consider a wing-aileron system having the degrees of freedom of vertical translation h, angular displacement α, and aileron rotation β. This system will be replaced in the analytical approach employed herein by the following system of infinite aspect ratio



where the chord and other dimensions are taken as those of a representative wing station. The stiffnesses of the linear spring and the two torsion springs are chosen to give natural frequencies which are the same as the bending, torsion, and aileron natural frequencies of the wing system.

It is shown in reference 10 that for a system of this type the flutter determinant is

$$\begin{vmatrix} \overline{A_{ch}} & A_{c\alpha} & A_{c\beta} \\ A_{ah} & \overline{A_{a\alpha}} & A_{a\beta} \\ A_{bh} & A_{b\alpha} & \overline{A_{b\beta}} \end{vmatrix} = 0$$

where

$$\overline{A_{ch}} = \Omega_h X - \mu + L_1 + iL_2$$

$$A_{c\alpha} = -\mu x_\alpha + L_3 + iL_4$$

$$A_{c\beta} = -\mu x_\beta + L_5 + iL_6$$

$$A_{ah} = -\mu x_\alpha + M_1 + iM_2$$

$$\overline{A_{a\alpha}} = \Omega_\alpha X - \mu r_\alpha{}^2 + M_3 + iM_4$$

$$A_{a\beta} = -\mu\left[r_\beta{}^2 - 2(x_1 - x_o)x_\beta\right] + M_5 + iM_6$$

$$A_{bh} = -\mu x_\beta + N_1 + iN_2$$

$$A_{b\alpha} = -\mu\left[r_\beta{}^2 + 2(x_1 - x_o)x_\beta\right] + N_3 + iN_4$$

$$\overline{A_{b\beta}} = \Omega_\beta X - \mu r_\beta{}^2 + N_5 + iN_6$$

The $\Omega_i X$ are functions of the structural properties of the wing and the flutter frequency ω through the relations

$$\Omega_h X = \mu(\omega_h/\omega)^2(1 + ig_h)$$

$$\Omega_\alpha X = \mu r_\alpha{}^2(\omega_\alpha/\omega)^2(1 + ig_\alpha)$$

$$\Omega_\beta X = \mu r_\beta{}^2(\omega_\beta/\omega)^2(1 + ig_\beta)$$

The L's, M's, and N's are aerodynamic force and moment coefficients defined and tabulated in reference 9 and 10 for the supersonic case, and are given for the sonic case in reference 11. The coefficients for subsonic compressible flow can be obtained for certain limited cases in references 7 and 8 and for incompressible flow from reference 12. A discussion of the significance of the structural parameters and of the numerical values assigned to them is given in a subsequent section.

The calculations to follow have been limited to the two subcases which involve two degrees of freedom, that is, flutter involving wing bending and aileron rotation, and flutter involving wing torsion and aileron rotation. It is recognized that, in cases of high coupling between bending and torsion, a three-degree-of-freedom analysis would probably be required to give a complete picture of the situation. In cases which are not highly coupled, however, consideration of the subcases should lead to satisfactory results, at least with regard to trends.

DISCUSSION OF PARAMETERS FOR THE CASES TREATED

The various parameters to be considered will be Mach number M, mass ratio parameter μ, aileron-hinge location x_1, aileron radius of gyration r_β^2, unbalance parameter x_β, frequency ratios ω_β/ω_h and $\omega_\beta/\omega_\alpha$, wing radius of gyration r_α^2, elastic-axis location x_o, and structural damping g_h, g_β. A table of values of these parameters employed in the calculations is presented as table I.

A variation in the mass ratio parameter μ may be considered as a variaton either in altitude or in wing mass. The values of $\mu = 15.708$ and 157.08 have been employed in the calculations and, for a particular configuration, might represent altitudes of sea level and approximately 50,000 feet, respectively.

The parameter x_1 is the coordinate of the aileron hinge relative to the leading edge of the wing. A value of $x_1 = 0.8$, for the hinge at 80 percent chord, has been used in the majority of the calculations. Some results are given for the hinge at 50 percent chord, or $x_1 = 0.5$.

The parameter r_β^2 as defined is related to the radius of gyration of the aileron referred to the hinge and depends in part on the ratio of aileron mass to wing mass and on the aileron mass distribution. For the case where $x_1 = 0.8$, if a ratio of aileron mass per unit length to wing mass per unit length of 1/10 is assumed, for a uniform mass distribution, r_β^2 would be approximately 0.005, whereas, for a triangular mass distribution, r_β^2 would be approximately 0.003. An average value of $r_\beta^2 = 0.004$ has been used in most of the calculations. For the case where $x_1 = 0.5$ the assumption of a mass ratio of 1/3 would lead to a similar average value for r_β^2 of about 1/10.

The parameter x_β is a reduced parameter which also involves the mass ratio and is related to the distance of the center of gravity of the aileron from the aileron hinge and represents the static unbalance of the control surface. Positive values of the parameter x_β repesent underbalance, whereas negative values represent overbalance. For $x_1 = 0.8$ and for the assumed ratio of aileron mass to wing mass, values of x_β ranging from -0.01 to 0.01 represent locations of the center of gravity of the aileron of about 20 percent of the aileron chord forward and rearward of the hinge. For $x_1 = 0.5$, the same variation in x_β would represent locations of the center of gravity within 5 percent of the hinge. It should be noted that in the torsion-aileron case, a complete static balance in the usual sense $(x_\beta = 0)$ is not quite sufficient to eliminate mass coupling. Actually, complete static balance against rotation implies $r_\beta^2 + 2(x_1 - x_o)x_\beta = 0$ so that a small amount of overbalance $(x_\beta < 0)$ is required. In the present case, with $r_\beta^2 = 0.004$, $x_1 = 0.8$, and $x_o = 0.4$, a value of $x_\beta = -0.005$ would be necessary to balance the aileron against rotation.

The parameter x_o gives the location of the elastic axis of the wing, whereas r_α^2 is the radius of gyration of the wing referred to the elastic axis. In the calculations for the torsion-aileron case the values $x_o = 0.4$ (elastic axis at 40 percent chord) and $r_\alpha^2 = 0.25$ have been employed.

DISCUSSION OF RESULTS

Results of the calculations are presented in figures 1 to 8. Figures 1 and 2 represent a general treatment with regard to Mach number. Generally, the regions of flutter are the areas bounded by loops, either open or closed. Where only a single curve is shown, the flutter region is the area below the curve. For these figures a particular set of parameters $\left(x_1 = 0.8,\ r_\beta^2 = 1/250,\ \mu = 157.08,\ g_h = 0, \text{ and } g_\beta = 0\right)$

has been selected and calculations performed for both the bending-aileron and torsion-aileron subcases for a range of Mach numbers from 0 to 2.0. Figures 1 and 2 present results for the coupled flutter of the wing-aileron system. Figure 3 presents some additional results for the incompressible case. Figure 4 presents results associated with single-degree-of-freedom flutter of the aileron. For figures 5 to 8 variations have been made in the parameters x_1, r_β^2, μ, g_h, and g_β and calculations performed at isolated Mach numbers to determine the effect of changes in these parameters. For convenience of reference to the figures and to show better the range of variables considered in the calculations, table I has been prepared.

For the figures relating to the bending-aileron case the ordinate is a parameter $b\omega_h/c$; the abscissa, the static unbalance parameter x_β or the frequency ratio ω_β/ω_h. For the torsion-aileron cases a parameter $b\omega_\alpha/c$ has been plotted against x_β or $\omega_\beta/\omega_\alpha$.

In figure 1 results are given for the bending-aileron case and are presented for various values of Mach number. The figures indicate the existence of a flutter region extending between an upper and lower value of the parameter $b\omega_h/c$ (at $M = 0$ it is expedient to employ $b\omega_h/v$). On the left side of figure 1, for which the flutter parameter is plotted against x_β for the case of zero restraint $(\omega_\beta = 0)$, the most striking result is the indication of a reverse effect of static mass balance at the transonic speeds. Whereas at $M = 2.0$ and also in the incompressible case, the flutter area lies almost entirely in the region of underbalance, as a Mach number of 1.0 is approached, the flutter region shifts to indicate flutter only for the condition of overbalance. It may be noticed, however, that the lower boundary of the flutter region appears at a rather high value of $b\omega_h/c$; this may be a fortunate circumstance in that practical values of this parameter may lie below the lower branch. On the right side of figure 1, the plots of the flutter parameter against frequency ratio ω_β/ω_h indicate that at a sufficiently high value of this frequency ratio, depending on the value of the unbalance parameter, flutter is completely eliminated.

The results given in figure 1 relate only to the coupled bending-aileron flutter. At transonic speeds additional roots appear which are associated with single-degree-of-freedom flutter of the aileron of the type discussed in reference 13. These roots are presented separately in a subsequent figure.

In figure 2 results are given for the torsion-aileron case and are presented in the form employed in figure 1. The results shown in the plots against x_β are generally comparable to those noted for the bending-aileron case in that flutter is indicated at transonic Mach numbers for the overbalanced aileron. In contrast to the results of the bending-aileron case, however, is the fact that flutter is indicated for the overbalanced aileron at $M = 0$ as well. This result for incompressible flow was further studied by varying the mass ratio parameter μ. Results are given in figure 3 and are discussed in a later paragraph. An attempt to study the transition from $M = 0$ to 0.7 could not be carried out because of the sparse nature of the available tables of coefficients. In the plots against frequency ratio, additional roots associated with flutter of the wing in pitch occur at transonic speeds. It might be recalled that various studies have been made for the case of a wing in pure torsion (see, for example, ref. 14) and have indicated that for certain axes of rotation and certain Mach numbers, flutter in this degree of freedom can occur. This type of flutter was encountered in the present calculations and is indicated by the essentially linear flutter boundaries at Mach numbers of 10/9, 10/8, and 10/7 (for the latter, at high values of $\omega_\beta/\omega_\alpha$ only). At high values of the frequency ratio these flutter boundaries approach the result of the solution which would be obtained by consideration of the single-degree-of-freedom (in pitch) equation.

As in the bending-aileron case, a sufficiently high value of the frequency ratio $\omega_\beta/\omega_\alpha$ serves to eliminate the coupled torsion-aileron flutter with the possible exception of those cases which appear to be identified with single-degree-of-freedom flutter. The required value of $\omega_\beta/\omega_\alpha$ is rather high, however, and it might be more practical or more desirable to eliminate the flutter by proper choice of $b\omega_\alpha/c$.

As in figure 1, the results associated with single-degree-of-freedom flutter of the aileron have not been presented and are discussed together with those of the bending-aileron case.

Because of the fact that, in the torsion-aileron flutter calculations at $M = 0$, flutter was indicated for a highly overbalanced aileron, this case has been given further consideration. An investigation of the effects of various parameters indicated that the range of values of the unbalance parameter x_β over which flutter could occur for the zero-frequency case was most strongly dependent on the mass ratio parameter μ. The results of calculations for various values of μ are shown in figure 3. A decrease in μ is seen to result in the division of the flutter area into two regions. As μ is further decreased, one of these regions either vanishes or recedes to greater negative values of x_β than were considered in the calculations. Flutter is still indicated, however, for a certain range of overbalance.

The results associated with single-degree-of-freedom flutter of the aileron which arose in connection with both the bending-aileron and torsion-aileron cases are presented in figure 4. The plots on the left show results obtained from solution of the coupled bending-aileron equations, while those on the right arose from the torsion-aileron case. Note that for the statically balanced aileron ($x_\beta = 0$) the two cases yield identical results at a given Mach number. This result is the same as that obtained from solution of the single-degree-of-freedom equation for flutter of the aileron. For values of x_β other than zero the coupled equations led to slightly different curves, thus some coupling with the wing motion is indicated. As mentioned previously, the pure single-degree-of-freedom flutter of a control surface has been treated in reference 13 where it is shown to be highly dependent on altitude, moment of inertia of the control surface, and structural damping.

Figures 5 to 8 give the results of some sample calculations made for other values of the control-surface parameters and the mass ratio parameter μ. Figure 5 presents results for the bending-aileron case for the values $\mu = 157.08$ and $x_1 = 0.8$, as treated in figure 1, with additional values of radius of gyration parameter ${r_\beta}^2$ of 1/180 and 1/60. Consideration of figure 5 and comparison with figure 1 indicates that an increase in ${r_\beta}^2$, corresponding to an increase in the moment of inertia of the aileron, at the higher Mach numbers tends to shift the flutter area more into the region of overbalance, thus the flutter area is extended over a wider range of values of the unbalance parameter. An increase in ${r_\beta}^2$ also extends the flutter region to slightly higher values of ω_β/ω_h. At $M = 10/9$ the major effect of an increase in ${r_\beta}^2$ is in narrowing the flutter region with respect to the parameter $b\omega_h/c$.

Figures 6(a) and 6(b) show results for the bending-aileron and torsion-aileron cases, respectively, for the set of control-surface parameters treated in figures 1 and 2 at a lower value of the mass ratio parameter μ. Comparison of figures 6(a) and 6(b) with the cases for lower mass ratio of figures 1 and 2 indicates that at the higher Mach numbers a decrease in the mass ratio results mainly in raising the values of $b\omega_h/c$ and $b\omega_\alpha/c$ which bound the flutter region. At $M = 10/9$ a decrease in the mass ratio shifts the flutter area farther into the region of overbalance.

Figure 7 shows results at three supersonic Mach numbers for the bending-aileron analysis of a large-chord control surface ($x_1 = 0.5$). For this case there is the same reverse effect of mass balance at transonic speeds as was noted for the case where $x_1 = 0.8$; and even at the higher Mach numbers the flutter area extends far into the region of overbalance. With regard to the plots against frequency ratio, the flutter region vanishes for values of ω_β/ω_h near unity.

Figure 8 treats the effect of structural damping for the bending-aileron case and for the set of control-surface parameters considered in figure 1. Comparison of figure 1 with figure 8 indicates beneficial effects of damping at $M = 10/7$ through elimination of the flutter region for the overbalanced condition and decreasing the value of ω_β/ω_h required to avoid flutter. At $M = 1.0$ the addition of structural damping shifts the flutter area slightly into the region of underbalance but, for the balanced aileron, only a very low value of ω_β/ω_h is required to eliminate flutter. At $M = 0.7$, the amount of damping considered eliminates the flutter region for the condition of overbalance and greatly reduces the value of ω_β/ω_h required to avoid flutter for the underbalanced aileron.

CONCLUDING REMARKS

Results have been presented of some sample calculations made for the bending-aileron and torsion-aileron flutter of a wing-aileron system of infinite aspect ratio for several values of Mach number in supersonic flow and also for some subsonic Mach numbers. It has been pointed out that the results obtained are based on the analysis of a highly idealized configuration and may be considerably altered by such factors as finite span, thickness, additional degrees of freedom, and other parameters which were not considered. It is of interest, however, to state the effects of certain of the parameters which seem to exert a major influence since they may serve to indicate trends.

With regard to mass balance of the control surface, the results indicate an apparent reverse effect of mass balance at the transonic speeds. In many of the calculations, flutter is indicated for an overbalanced aileron near a Mach number of 1, whereas, at higher and lower Mach numbers, overbalance tends to eliminate flutter. In practice, however, it should be possible to avoid flutter for the overbalanced condition by an appropriate choice of wing stiffnesses.

With regard to aileron frequency, a high value of the ratio of aileron frequency to wing bending or torsional frequency appears beneficial in eliminating coupled wing-aileron flutter and, in fact, a sufficiently high value will eliminate it, except in those cases which appear to be identified with flutter of the wing in pure pitch. In many cases, however, the value of the frequency ratio required to avoid flutter appears to exceed unity and is probably not easy to attain.

Structural damping of the wing and aileron appears to be generally beneficial in reducing the amount of balance required to avoid flutter or in reducing the required value of the ratio of aileron frequency to wing frequency.

Near a Mach number of 1 the possibility exists of flutter of the aileron in a single degree of freedom. The sequence of roots in a particular calculation may be confused by the appearance of roots associated with this flutter. This type of flutter has been treated elsewhere and can probably be eliminated by the proper choice of such parameters as moment of inertia of the aileron.

Although certain parameters seem to give beneficial results, it appears difficult to arrive at any single practical criterion for the prevention of flutter involving control surfaces at supersonic speeds. Further work on the subject seems obviously needed since for application to a particular design the effects of parameters not considered will have to be taken into account before more specific conclusions can be drawn.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., November 9, 1953.

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TABLE I

RANGE OF VARIABLES CONSIDERED IN THE CALCULATIONS

Figure	Case	x_β	$\omega_\beta/\omega_\alpha$ or ω_β/ω_h	μ	x_1	r_β^2	x_o	r_α^2	g_h, g_β	M
1	h,β	Varied	0	157.08	0.8	1/250	---	----	0,0	2.0, 10/7, 10/9, 1.0, 0.7, and 0
	h,β	0.01 0 -.01	Varied	157.08	.8	1/250	---	----	0,0	2.0, 10/7, 10/9, 1.0, 0.7, and 0
2	α,β	Varied	0	157.08	.8	1/250	0.4	0.25	0,0	2.0, 10/7, 10/8, 10/9, 1.0, 0.7, and 0
	α,β	.01 0 -.01	Varied	157.08	.8	1/250	.4	.25	0,0	2.0, 10/7, 10/8, 10/9, 1.0, 0.7, and 0
3	α,β	Varied	0	157.08 78.54 39.27 15.71	.8	1/250	.4	.25	0,0	0
4(a)	h,β	.01 0 -.01	Varied	157.08	.8	1/250	---	----	0,0	10/8, 10/9, and 1.0
4(b)	α,β	.01 0 -.01	Varied	157.08	.8	1/250	.4	.25	0,0	10/8, 10/9, and 1.0
5	h,β	Varied	0	157.08	.8	1/180 and 1/60	---	----	0,0	2.0, 10/7, and 10/9
	h,β	.01 0 -.01	Varied	157.08	.8	1/180 and 1/60	---	----	0,0	2.0, 10/7, and 10/9
6(a)	h,β	Varied	0	15.708	.8	1/250	---	----	0,0	10/7, and 10/9
	h,β	0 -.01	Varied	15.708	.8	1/250	---	----	0,0	10/7, and 10/9
6(b)	α,β	Varied	0	15.708	.8	1/250	.4	.25	0,0	2.0, and 10/9
	α,β	0 -.01	Varied	15.708	.8	1/250	.4	.25	0,0	2.0, and 10/9
7	h,β	Varied	0	157.08	.5	1/10	---	----	0,0	2.0, 10/7, and 10/9
	h,β	.01 0 -.01	Varied	157.08	.5	1/10	---	----	0,0	2.0, 10/7, and 10/9
8	h,β	Varied	0	157.08	.8	1/250	---	----	0.03,0.1	10/7, 1.0, and 0.7
	h,β	.01 0 -.01	Varied	157.08	.8	1/250	---	----	.03, .1	10/7, 1.0, and 0.7

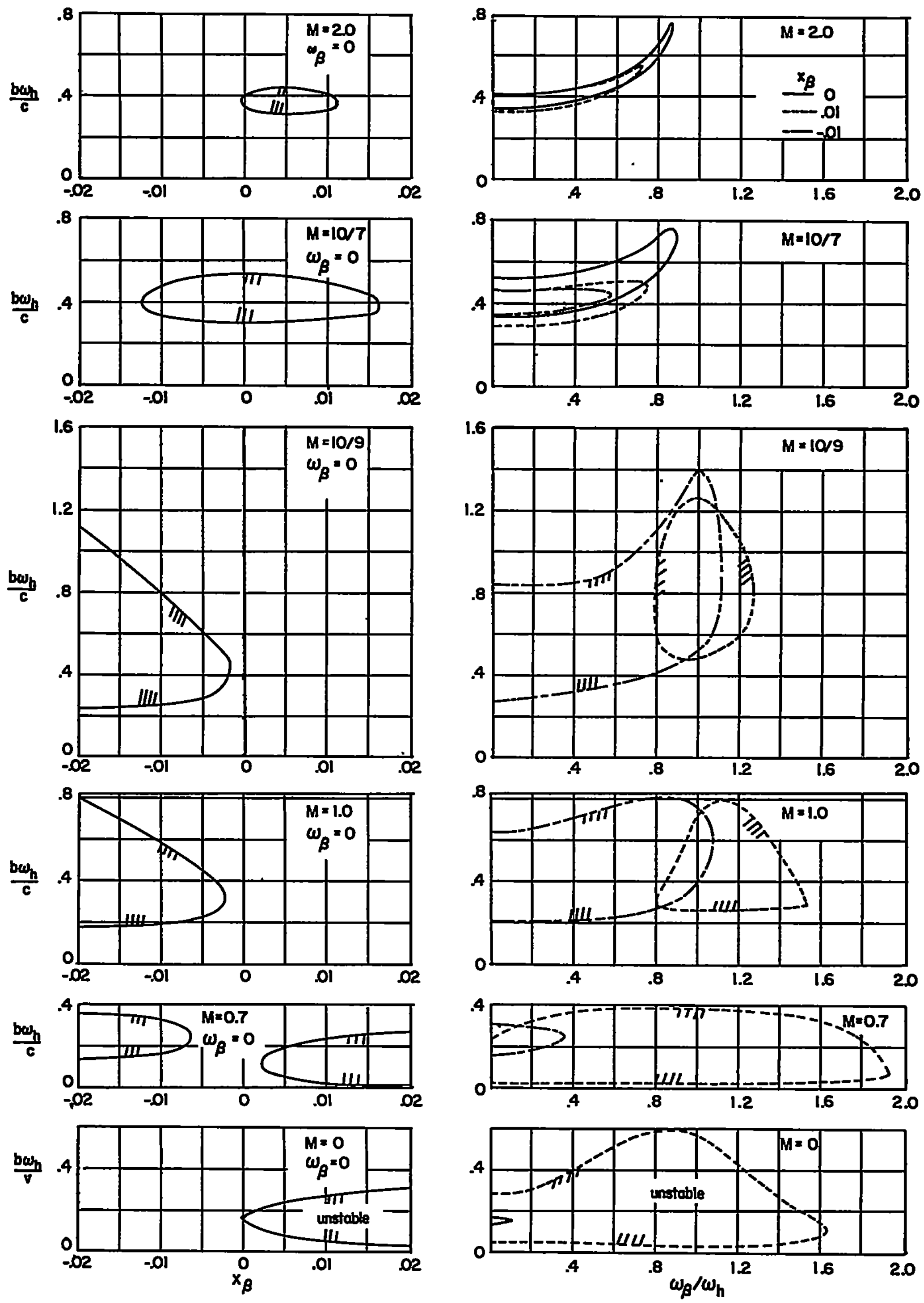


Figure 1.- Flutter parameter $b\omega_h/c$ or $b\omega_h/v$ against unbalance parameter and frequency ratio for bending-aileron case. $\mu = 157.08$; $x_1 = 0.8$; $r_\beta{}^2 = 1/250$.

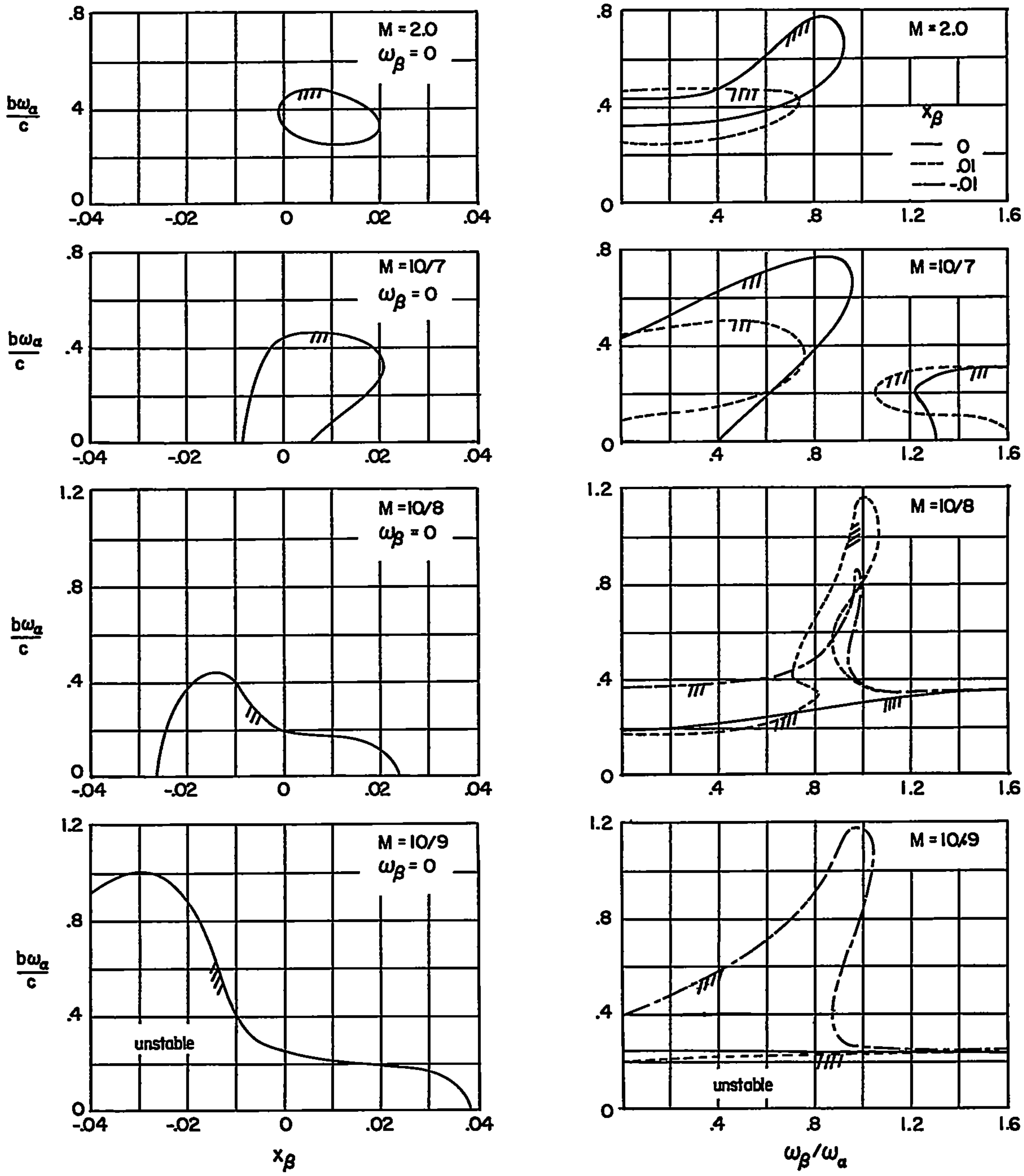


Figure 2.- Flutter parameter $b\omega_\alpha/c$ or $b\omega_\alpha/v$ against unbalance parameter and frequency ratio for torsion-aileron case. $\mu = 157.08$; $x_o = 0.4$; $x_1 = 0.8$; $r_\alpha{}^2 = 0.25$; $r_\beta{}^2 = 1/250$.

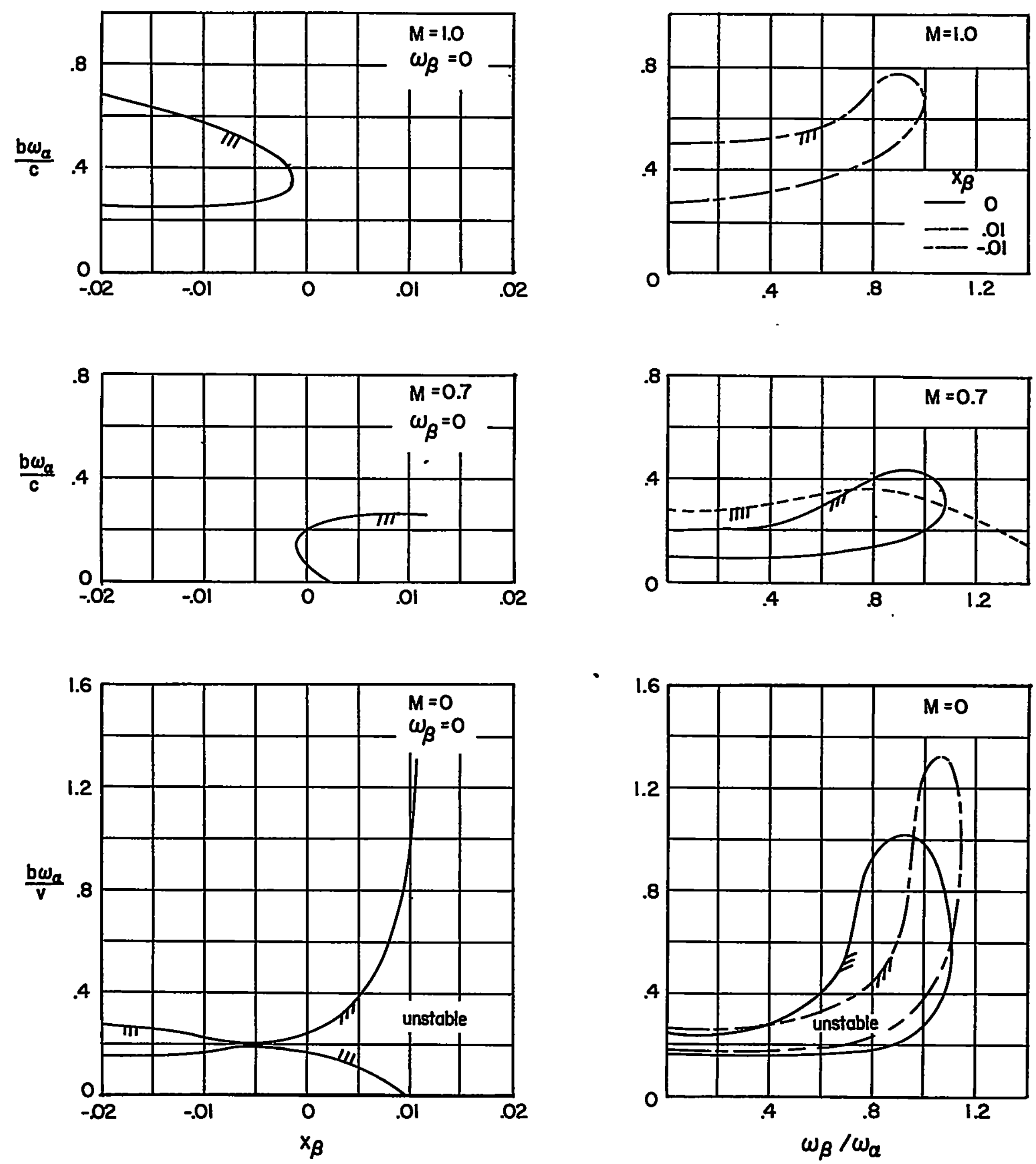


Figure 2.- Concluded.

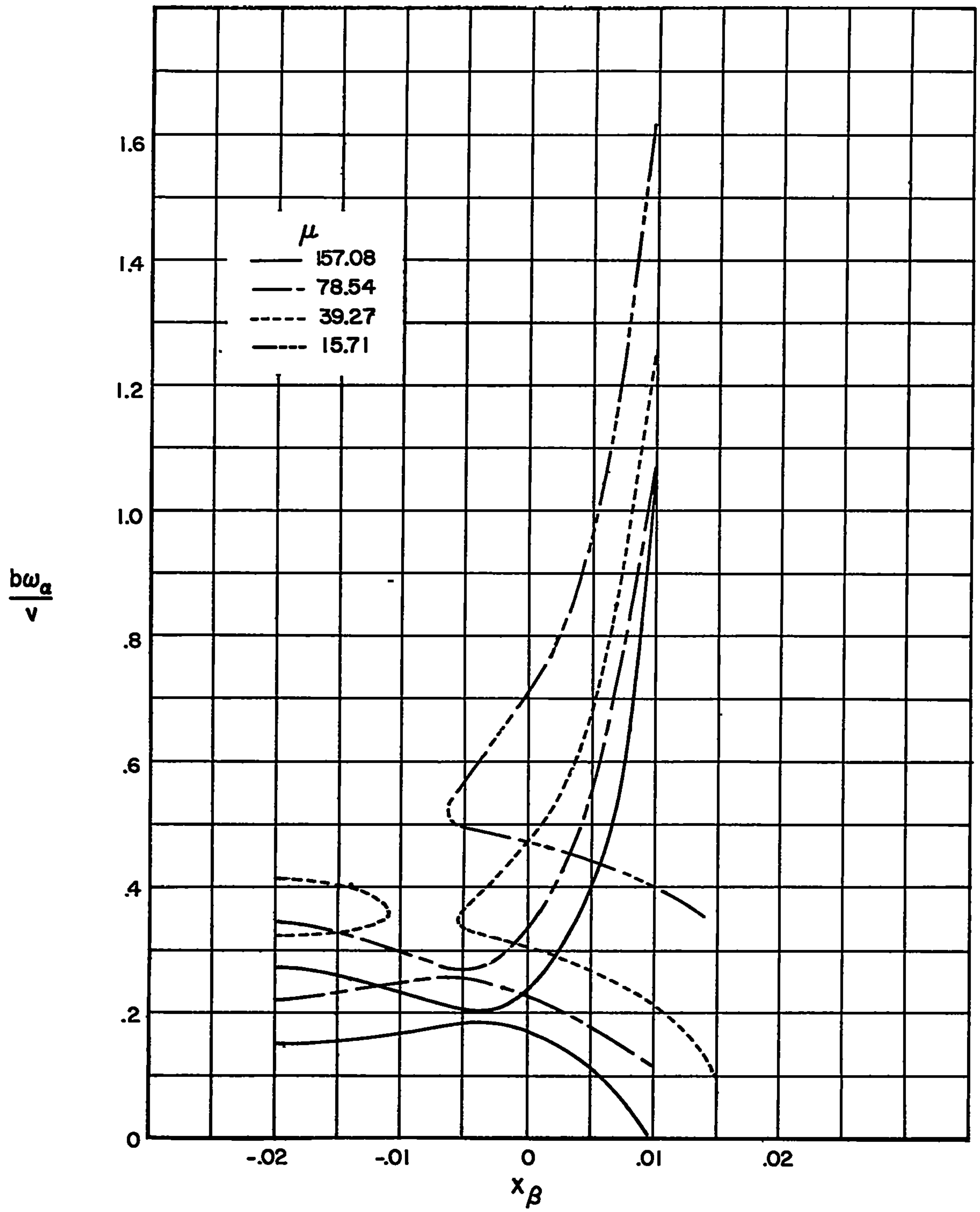
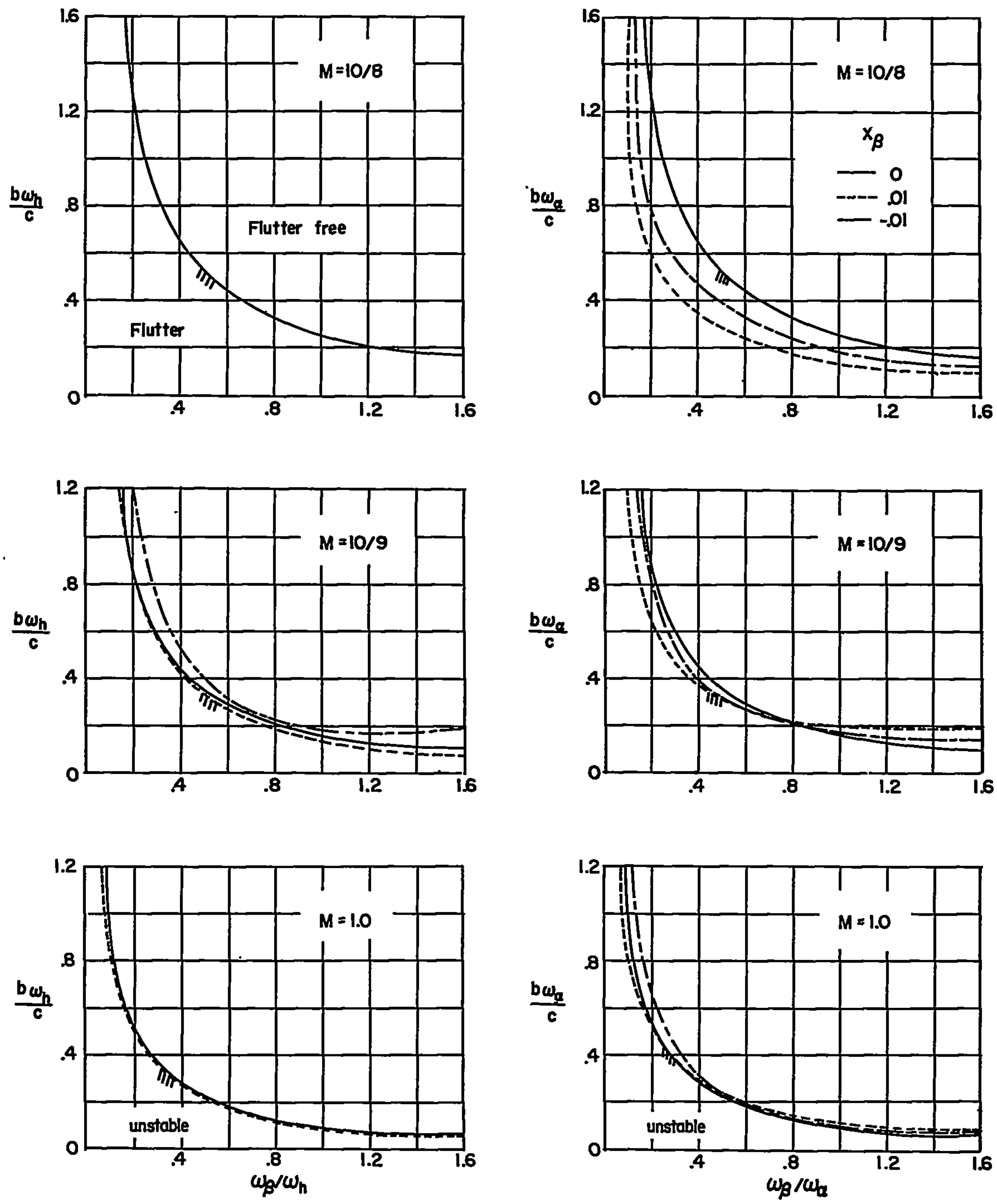


Figure 3.- Flutter parameter $b\omega_\alpha/v$ against unbalance parameter for torsion-aileron case for $M = 0$. $r_\beta{}^2 = 1/250$; $\omega_\beta = 0$; $x_1 = 0.8$; $x_0 = 0.4$; $r_\alpha{}^2 = 0.25$; various values of mass ratio parameter μ.



(a) Bending-aileron case.
$\mu = 157.08$; $x_1 = 0.8$;
$r_\beta{}^2 = 1/250$.

(b) Torsion-aileron case.
$\mu = 157.08$; $x_1 = 0.8$;
$r_\beta{}^2 = 1/250$; $x_o = 0.4$;
$r_\alpha{}^2 = 0.25$.

Figure 4.- Flutter parameter against frequency ratio for roots associated with single-degree-of-freedom flutter of the aileron.

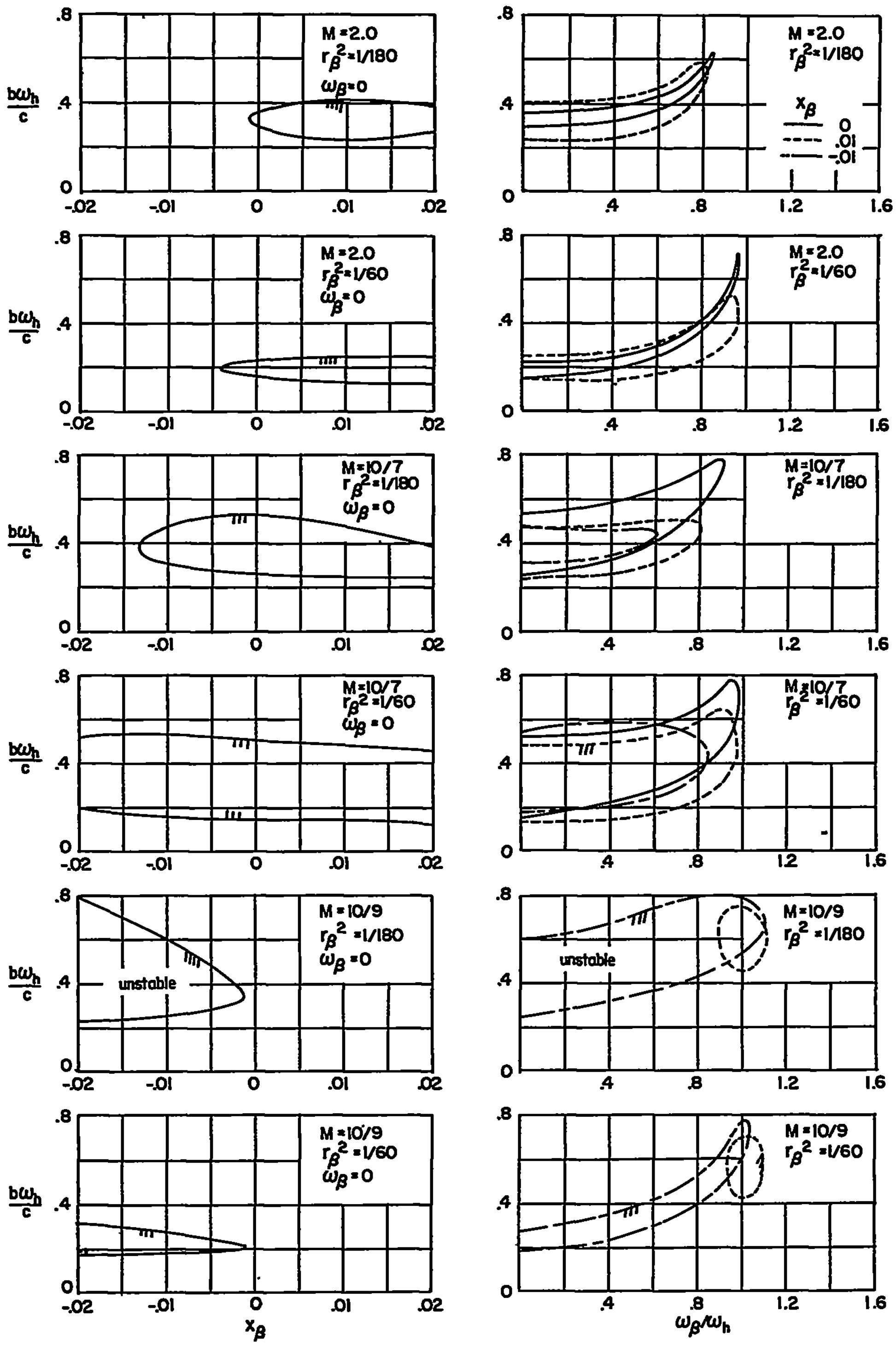
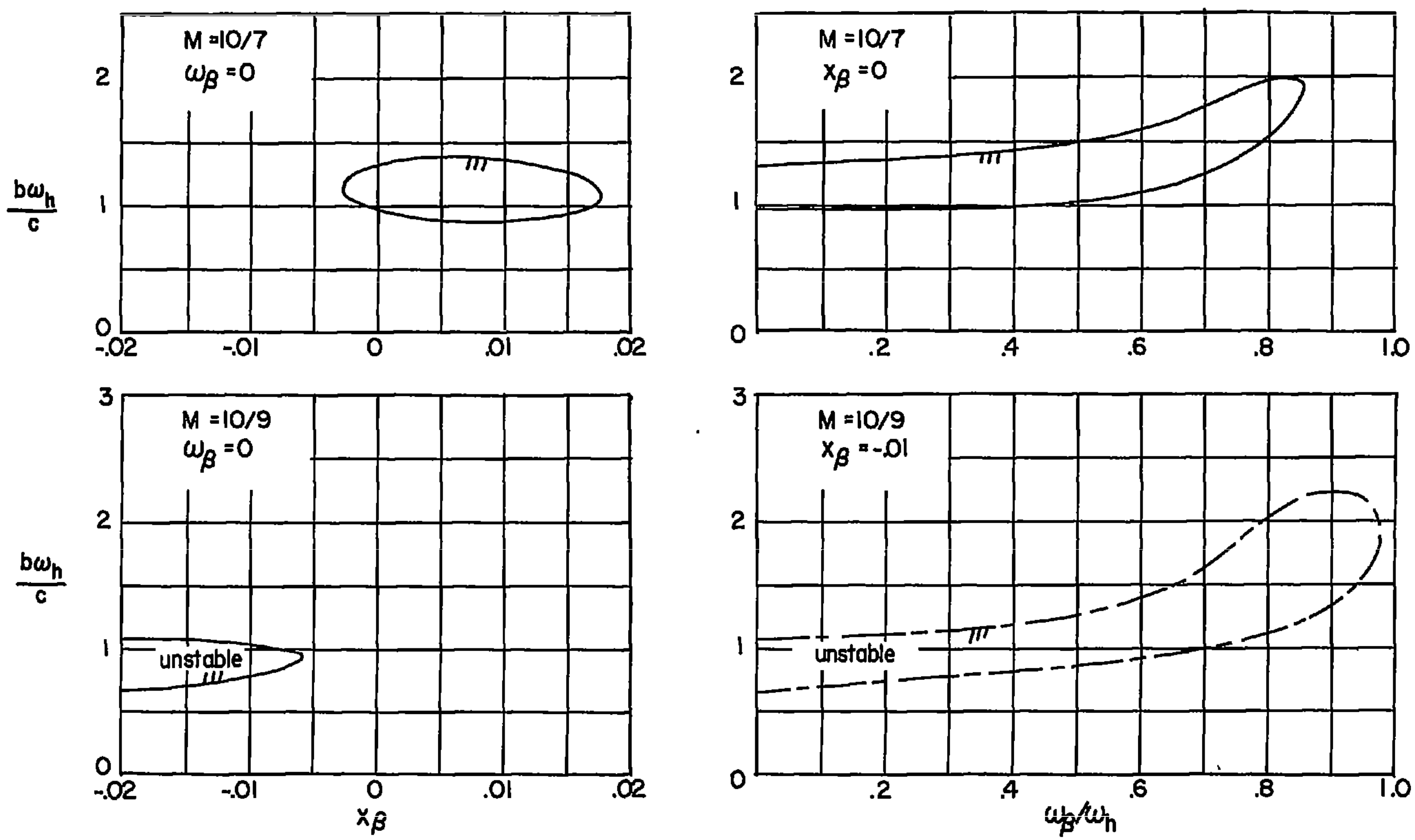
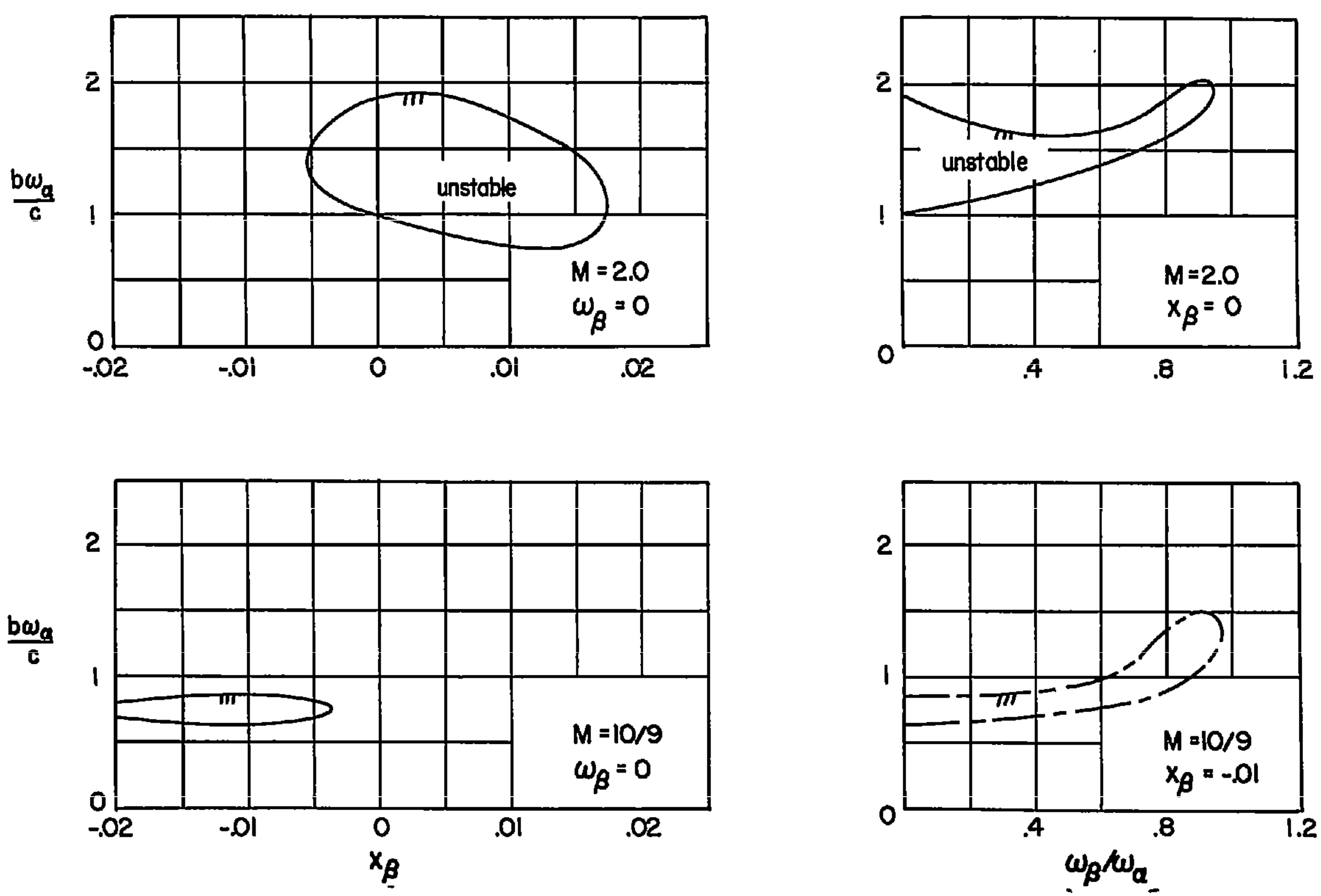


Figure 5.- Flutter coefficient $b\omega_h/c$ against unbalance parameter and frequency ratio for bending-aileron case. $\mu = 157.08$; $x_1 = 0.8$; $r_\beta^2 = 1/60$ and $1/180$.



(a) Bending-aileron case. $\mu = 15.708$; $x_1 = 0.8$; $r_\beta^2 = 1/250$.

Figure 6.- Flutter parameter $b\omega_h/c$ and $b\omega_\alpha/c$ against unbalance parameter and frequency ratio for bending-aileron case and torsion-aileron case.



(b) Torsion-aileron case. $\mu = 15.708$; $x_o = 0.4$; $x_1 = 0.8$; $r_\alpha^2 = 0.25$; $r_\beta^2 = 1/250$.

Figure 6.- Concluded.

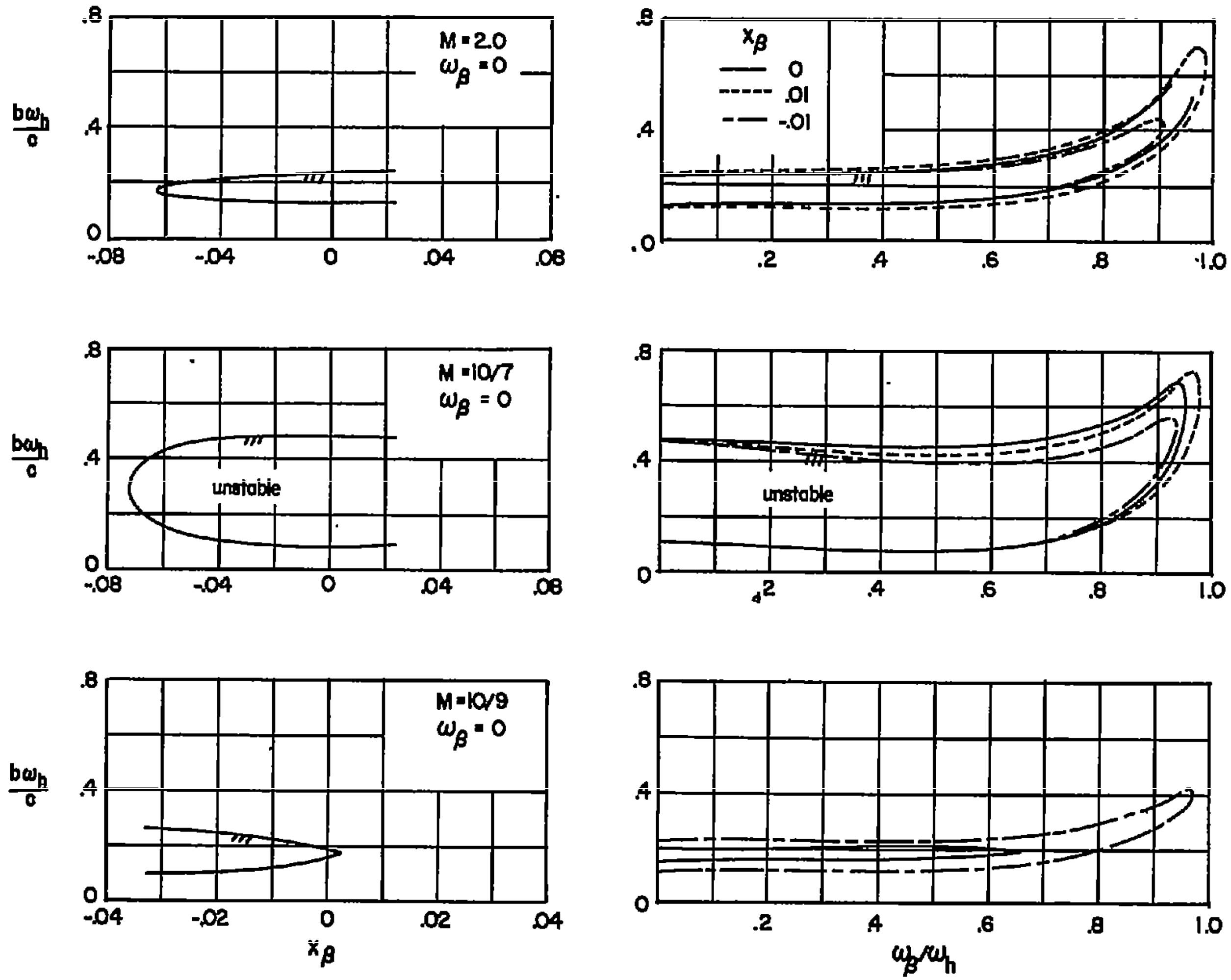


Figure 7.- Flutter parameter $b\omega_h/c$ against unbalance parameter and frequency ratio for large-chord control surface. Bending-aileron case. $\mu = 157.08$; $x_1 = 0.5$; $r_\beta^2 = 1/10$.

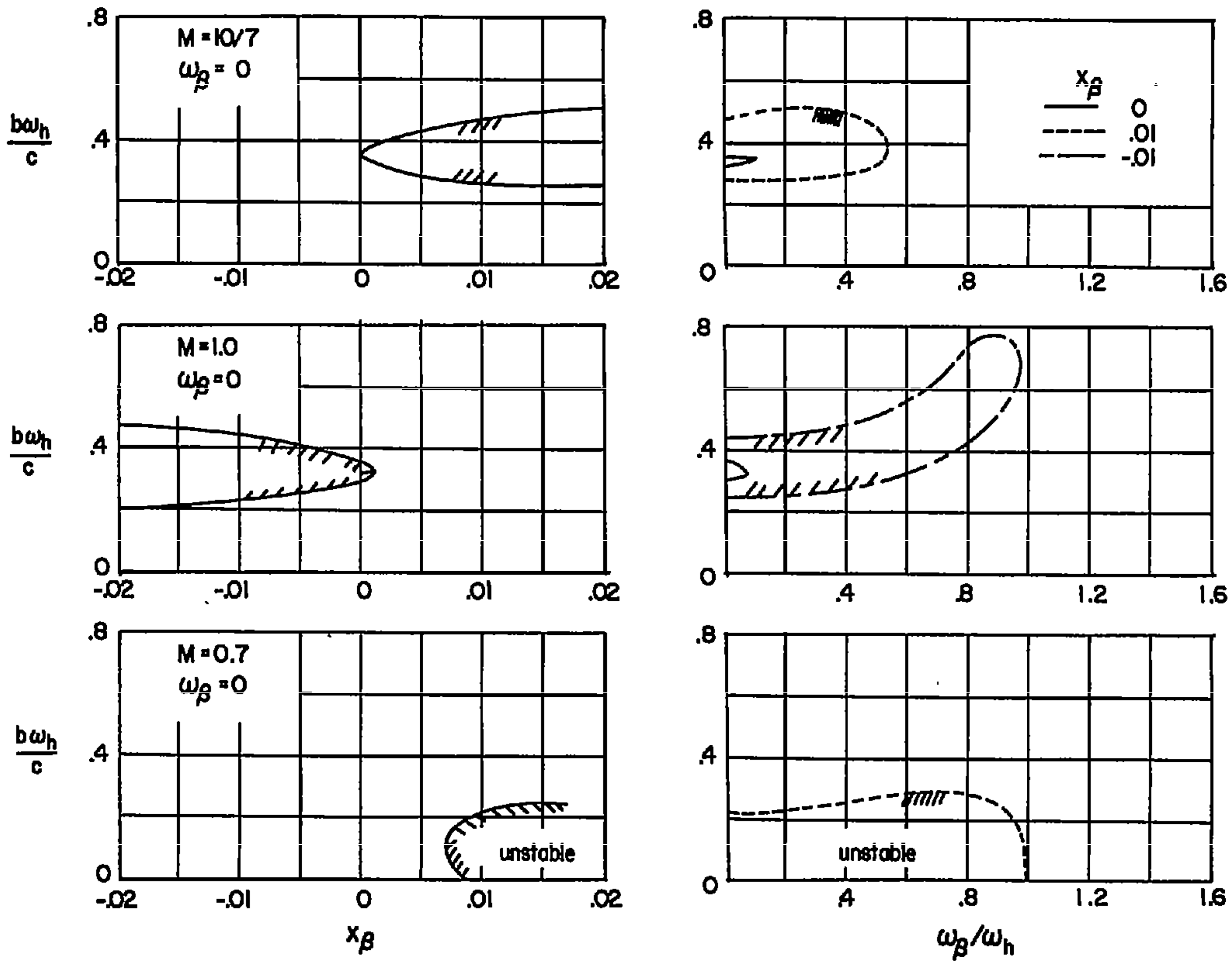


Figure 8.- Flutter parameter $b\omega_h/c$ against unbalance parameter and frequency ratio. Bending-aileron case with structural damping. $\mu = 157.08$; $r_\beta^2 = 1/250$; $g_h = 0.03$; $g_\beta = 0.1$; $x_1 = 0.8$.

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